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Solution for System of Fractional Order Wiener-Hope Dynamical System and System of Nonlinear Variational Inequality Problem

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Abstract—*The aim of this paper is to introduce a new system of Wiener - Hopf equation (SWHE) defined on a real Hilbert space. We study the system of nonlinear variational inequality problem on real Hilbert space. we consider a system of new fractional order Wiener-Hopf dynamical system (SFOWHDS) for system of nonlinear variational inequalities problem (SNVIP) using the Wiener-Hopf equations technique. Moreover, the existence of a solution to such a fractional order Wiener -Hopf dynamical system is considered and there is demonstrated a systemic solution to such a dynamical system. We show that the solution of system of fractional order Wiener-Hopf dynamical system is exist and unique. This type dynamical system is interesting to study because it can be apply in the various real world problems.*

Keywords: *Variational inequality problem, fractional derivative, Wiener- Hope equation, projected dynamical system, Lipschitz continuous mapping, non-expansive mapping, exponentially stability.*

INTRODUCTION

Integer order differential and integral equations (IDEs) make up the majority of the mathematical models. Since a few decades ago, non-integer order differential equations (FDEs) have allowed for the more accurate and precise formulation of actual events. Many researchers have grown passionate in the study of fractional differential system dynamics in recent years, and many interesting and significant outcomes, which include factional-order differential systems having chaos have been reported. Recently, For the purpose of learning to use fractional calculus, Nonlinear system stability analysis has been enhanced. The use of fractional calculus to model nonlinear systems served as an inspiration, these studies used the integer-order stabilisation approach.

The direct approach of fractional Lyapunov are suggested by the author in an effort to extend our understanding of fractional calculus and system theory. The use of fractional calculus in reality is made practical and inexpensive by quicker processing and less expensive memory. [Chen, [8]]. There are various area like informatics and material, control of fractional order dynamical system. In some cases, a fractional-order controller for a non-integer order system may perform better in terms of transient response than a traditional integer-order controller. Modern calculus is the generalization of classical integer-order calculus. Important uses in the sciences of mechanics, viscoelasticity, signal processing, economics, optimization, oceanography, bacteria that randomly move through fractal materials in search of food, neurons modelling, chaotic systems and others as well. It is significant to highlight that fractional differential systems can be used to explain a wide range of physical phenomena that include memory and inherited characteristics. For more read, we go to references [9]- [12].

In 2014, Zeng at.al. [14] studies at a class of global non-integer order projective dynamical systems and demonstrating the existence and originality of this kind of system's solution. With regard to these dynamical systems, it is possible to establish whether the equilibrium point exists and with the suitable conditions, its α-exponential stability.

Stampacchia initially proposed the variational inequality problem in 1964 [1], whose definition is as below:

Let C be a non-empty subset of Hilbert space H which is closed and convex and let consider nonlinear mapping T from subset C to H. The typical VIP is then introduced in the manner described below:

$\langle T(x^*), x - x^* \rangle \geq 0,$for all$x \in C.$ (1.1)

Variational inequality problem (VIP) is the name given to equation (1.1) and indicated by VI(C,T) and the collection of all solution of (1.1) is indicated by Ω(VI(C,T)), that is,

$\Omega(VI(C,T)) = \{x^* \in C: \langle T(x^*), x - x^* \rangle \geq 0, \forall x \in C\}.$

The collection of all T's fixed point is indicated by Fix(T). It is well know results that VIP (1.1), which is outlined as the fixed point problem (FPP) that follows:

find x^* in C such that $x^* = P_C(I - \mu T)x^*$. (1.2)

where P_C is refer best approximation operator. from Hilbert space H to C, where $\mu > 0$ is non-negative constant and I stand for mapping from H on to H, which is identity. If the mapping T is η-strongly monotone and κ-Lipschitzian, then the operator $P_C(I - \mu T)$ is a contraction on subset C if $0 < \mu < 2\eta/\kappa^2$. The Banach Contraction principle in this situation ensures that equation (1.1) has exactly one solution x^* in C. Sequence is described as

$x_{n+1} = P_C(I - \mu T)x_n, \forall n \in \mathbb{N}$, (1.3)

converges x^* in C is known as The Picard iteration method's. This process is also called projection gradient method (PGM) (see [2]). Stampacchia studied the problem of variational inequality which widely use in field of mechanics. Moreover variational inequality is a one of the power full tool to studying different problem which are related to different branches of pure and applied mathematics. It is very useful in field of differential equation mechanics, transportation problem, operation research, control problem, equilibrium problem, fuzzy controls system and networking related problem. many authors use the concept of projection gradient method (PGM) in different ways, (see [3] [15] [16] [17]). This all technique are used in diverse area of science and being productive and innovative. This tech- nique are motivate to generalized the problem and extends the concept of variational inequality and convex optimization problem.

In 2001, Verma [17] presented the generalized variational inequality problem system, which studied as below:

Let $T: H \to H$ be the operator be nonlinear and C be a convex and closed subset of Hilbert space H that is not empty, to find $x^*, y^* \in C$,such that

$$\begin{cases} \langle \rho T(x^*) + y^* - x^*, x - y^* \rangle & \geq 0, \text{for all } x \in C, \\ \langle \eta T(y^*) + x^* - y^*, x - x^* \rangle & \geq 0, \text{for all } x \in C. \end{cases} \quad (1.4)$$

here ρ, η > 0 be constant. In 2001, Verma [17] Some algorithmic methods involving converges analysis for roughly addressing the VIP has been proposed. Convex optimisation problems and various other linear and nonlinear variational inequality problems are resolved as well using the projected dynamical system (for more information, see [18, 20]). In 1993, D. Zhang and A. Nagurney [21] introduce the Dynamical system and Variational inequality problem both and further in 1996 Further they studied about Projected dynamical system and VIP and provide some important results.

Noor [24] investigated the fixed point formulation in 2003 for Evaluation of the differential equation for quasi type VIP is the goal. Numerous dynamical systems recognised and proposed by Dupuis and Nagurney [21] are included in this dynamical system and Friesz et al. [23].

Cojocaru et al. [4] in 2005, A Lipschitz continuous operator on every Hilbert space of finite dimensional, for any nonempty convex and compact set, evolutionary projected dynamical systems, and variational inequality problem were explored, and they demonstrated the solution to this sort of problem.

The topic of dynamical systems has drawn the interest of several authors, who have written in these publications as a consequence of their thorough investigation. (see [21] [26] [28] [20] and the references therein).

On the other hand, In 1991, P. Shi [16] introduced the Wiener-Hopf equation and In 1992, Robinson [35] also studied Wiener-Hopf equation independently and using the projection technique. Wiener - Hope equation define as follows:

Consider no-void, closed and convex subset C of real Hilbert space H and T be a nonlinear operator from C to H, We view that problem as finding $x \in H$ such that

$Q_C x + \rho T P_C x = 0$, (1.5)

where $\rho > 0$ be constant and $Q_C = I - P_C$ substantiate the Wiener-Hope equation's equality with the variational inequalities. This show that solution of Wiener-Hope equation and solution of variational inequality problem can obtain if one of them exist and also unique. In 1993, Noor [36] show that generalized Wiener - Hope equation is equivalent to the variational inequity problem. In 2002, Noor [38] established the Wiener-Hopf equations method to analyse a dynamical system for variational inequality and to demonstrate the dynamical system's global asymptotic stability. The Wiener - Hopf dynamical system has global asymptotically stability property for pseudomonotone operator. In 2007, Noor and Zhenyu Huang consider about the types of nonlinear and non-expansive operators utilised by the new class of Wiener Hopf equations.In 2010, Guanghui Gu and Yongfu Su [39] studied approximations of the Wiener-Hopf equation and generalised variational inequality problem. In 2013, Changun Wu [40] give theory to find the solution of Wiener - Hopf equation and common solution of variational inequality and sand a collection of non-expansive mapping's fixed points under some condition.

Section 2 of this paper offers preliminary information, while Section 3 contains the major fact which show the solution of system of fractional order Wiener Hopf Projected dynamical system are exist and unique. Section 4 contains conclusion of this paper.

PRELIMINARIES

Firstly, we introduce some definition and lemma which are useful.

Definition 2.1 A nonlinear operator T from C to H is called

(1) monotone if

$$\langle Tx - Ty, x - y \rangle \geq 0, \textit{for all } x, y \in C,$$

(2) η-strongly monotone if $\exists\, \eta > 0$ such that

$\langle Tx - Ty, x - y\rangle \geq \eta \parallel x - y \parallel^2$ *,for all* $x, y \in C$,

(3) β-Lipschitzian if $\exists\, \beta > 0$ such that

$\parallel Tx - Ty \parallel \leq \beta \parallel x - y \parallel$ *,for all* $x, y \in C$,

(4) Non-expansive if

$\parallel Tx - Ty \parallel \leq \parallel x - y \parallel$ *,for all* $x, y \in C$,

(5) Contraction if ∃ k ∈ [0, 1) s. t.

$\parallel Tx - Ty \parallel \leq k \parallel x - y \parallel$ *,for all* $x, y \in C$.

Let $x \in H$ be a element not belong to subset C. A point $z \in C$ is said to be a nearest point to x if $d(x, C) = \parallel x - y \parallel$. The set of all best approximations from x to C, which may or may not be empty, is denoted by

$P_C(x) = \{y \in C: d(x, C) = \parallel x - y \parallel\}$ (2.1)

Consider the closed, convex, nonempty subset C of the set H. Then, for any element $x \in H$, there exist a unique best approximation point (nearest point) $P_C(x)$ of C such that

$\parallel x - P_C(x) \parallel \leq \parallel x - y \parallel$ *,for all* $y \in C$. (2.2)

Note that P_C is non-expansive from H onto C. Lemma 2.1 [29] Given $x \in H$, $z \in C$, Then $P_C(x) = z$ if and only if $\langle x - z, z - y\rangle \geq 0$*,for all* $y \in C$.

Proposition 2.1. [30] For any element $x \in C$ and any $v \in C$ the limit $\prod_C(x, v) = \lim_{\delta \to 0^+} \frac{P_C(x+\delta v) - x}{\delta}$,

exits and $\prod_C(x, v) = P_C(v)$.

Definition 2.3. [30] Let H be a Hilbert space having any possible dimensions and $C \subseteq H$ be a closed, convex, and nonempty subset. Let F be only one-valued mapping on C. Then the differential equation

$\frac{dx(t)}{dt} = \prod_C(x(t), -F(x(t))), x(0) = x_0 \in C$ (2.3)

is said to be the F and C-related projected differential equation. Then a solution to (2.3) is x(t) an absolutely continuous function if $x: [0, T) \subseteq R \to H$ with $x(t) \in C$, ∀ t∈ $[0, T)$ and $dx/dt = \prod_C(x(t), -F(x(t)))$, for almost every $t \in [0, T)$.

To corroborate our conclusions regarding the concepts of stability in dynamic systems, the following definitions and lemma are important.

Take note of the general differential equation

$\frac{dx}{dt} = f(x(t))$, (2.4)

Definition 2.5 [31]

If $f(x^*) = 0$ then x^* point is referred to as the equilibrium point of equation (2.4).

If for any $\varepsilon > 0$, $\exists \delta > 0$ such that, for any $x_0 \in B(x^*, \delta)$ the solution x(t) of the differential equation with initial point $x(0) = x_0$ exists and $x(t) \in B(x^*, \varepsilon)$ (2.5) for all t > 0, then an equilibrium point x^* of (2.4) called stable.

Lemma 2.2. [33] (Gronwall Lemma) Let $\hat{u}$ and $\hat{v}$ be continuous real-valued functions with a domain $\{t: t \geq t_0\}$ and let $\alpha(t) = \alpha_0(|t - t_0|)$, where α_0 be monotone non-decreasing function. If for all $t \geq t_0$,

$\hat{u}(t) \leq \alpha(t) + \int_{t_0}^{t} \hat{u}(s)\hat{v}(s)ds$. (2.6)

Then $\hat{u}(t) \leq \alpha(t) e^{\int_{t_0}^{t} \hat{v}(s)ds}$. (2.7)

Lemma 2.3. [38] The VIP (1.1) have solution $x^* \in C$ iff the Wiener - Hopf equation (1.5) have unique solution $u \in H$ where

$x = P_C u$, (2.8)

$u = x - \rho Tx$. (2.9)

Using the equation (2.8) and (2.9), the Wiener - Hope equation can be written as

$x - \rho Tx - P_C[x - \rho Tx] + \rho T P_C[x - \rho Tx] = 0$. (2.10)

Using above equivalence, Noor analyze a new system associate with VIP (1.1) as follows:

$\frac{dx}{dt} = \lambda\{P_C[x - \rho Tx] - \rho T P_C[x - \rho Tx] + \rho Tx - x\}$.

(2.11)

with $x(t_0) = x_0$ and λ is constant. Equation (2.11) is known as Wiener - Hopf dynamical system.

Now, let's think about new dynamical system:

$$D_\omega^\alpha u(t) = \gamma\{P_C(u(t) - \rho Tu(t)) - \rho T P_C(u(t) - \rho Tu(t)) + \rho T(u(t)) - u(t)\},$$

(2.12)

where $\alpha \in (0,1)$ and γ is a constant related to VIP (1.1). The system (2.12) is called fractional order Wiener-Hopf dynamical system (FOWHDS) associated with a VIP (1.1).

Definition 2.9. [45] Riemann-Liouville definition of non-integer derivative of order $\alpha \in \mathbb{R}$, of u(t) is described as:

$I_{t_0}^\alpha u(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t - \tau)^{\alpha - 1} u(\tau) d\tau, t > t_0$, (2.13)

where the Euler gamma function is denoted by Γ.

Definition 2.10. [45] The Caputo derivative of non-integer derivative of order $\alpha \in \mathbb{R}_+$ of function $u(t) \in \mathbb{C}^n, ([t_0, +\infty], \mathbb{R})$ is given by

$$D_{t_0}^{\alpha}u(t) = I_{t_0}^{n-\alpha}u^{(n)}(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}u^{(n)}(\tau)d\tau, t > t_0,$$

(2.14)

where n is an integer that is positive so that $\alpha \in (n-1, n)$.

Definition 2.12. [47] If the dynamical system's (2.12) any two solutions u(t) and v(t) with distinct beginning point by u_0 and v_0 satisfy the condition

$$\| u(t) - v(t) \| \leq \eta \| u_0 - v_0 \| e^{-\lambda t\alpha}, \forall t \geq t_0,$$

then the system (2.12) is called α-exponentially stable with degree λ.

Lemma2.4 [48] Let $n \in \mathbb{Z}_+$ and $n-1 < \alpha < n$. $u(t) \in \mathbb{C}^n[a,b]$, then

$$I_t^{\alpha}D_t^{\alpha}u(t) = u(t) - \sum_{k=0}^{n-1}\frac{u^k(a)}{k!}(t-a)^k.$$

In particular, if $0 < \alpha < 1$ and $u(t) \in C^1[a,b]$. $I_t^{\alpha}D_t^{\alpha}u(t) = u(t) - u(a)$. (2.15)

Lemma 2.5. [47] Consider a function, which is a continuous on $[0, +\infty)$ and satisfies

$D_t^{\alpha}u(t) \leq \theta u(t)$, (2.16)

where $0 < \alpha < 1$ and θ is a constant. Then

$u(t) \leq u(0).exp\left(\frac{\theta t^{\alpha}}{\Gamma(\alpha+1)}\right)$.

Lemma 2.6. [47]- [49] Consider the system

$D_t^{\alpha}u(t) = g(t, u(t)), t > t_0$, (2.17)

with in initial condition $u(t_0)$, where $0 < \alpha \leq 1$and $g: [t_0, \infty) \times C \to H, C \subset H$. If $g(t, u(t))$ be locally Lipschitz continuous with regard to u(t), then ∃ unique solution of (2.17) on $[t_0, \infty) \times C$.

Lemma2.7. [49] With respects to the continuous function with real values g(t , u(t)), mentioned in (2.17), we have

$\| I_t^{\alpha}g(t, u(t)) \| \leq I_t^{\alpha} \| g(t, u(t)) \|$,

Where $\alpha \geq 0$ and $\|\cdot\|$ indicates an arbitrary norm.

MAIN RESULTS

First we discuss about some important Lemma and results: In 2001, Verma [17] present following lemma:

Lemma 3.1. [17] Solution of problem (1.4) are x^* and y^* iff

$y^* = P_C(x^* - \rho T x^*)$ and $x^* = P_C(y^* - \eta T y^*)$, (3.1)

where ρ, η be a positive constant.

The VIP (1.4), is similar to the system of Wiener-Hopf equations is now being considered. Let T be a nonlinear mapping from Hilbert space H to itself and $\rho, \eta > 0$ be constant, we regard this problem to be to identify x^*, y^*, u^*, v^* in H such that

$$\begin{cases} Q_C(v^*) + \rho T P_C(u^*) &= y^* - x^*, \\ Q_C(u^*) + \eta T P_C(v^*) &= x^* - y^*, \end{cases} \quad (3.2)$$

Where $Q_C = I - P_C$ where I be an identity operator on H.

In 2018, Narin Petrot and Jittiporn Tangkhawiwetkul [42] present the lemma, which show the equivalence of the problem (1.4) and (3.2).

Lemma 3.2. [42] Let $T: H \to H$ be a continuous Lipschitz mapping. There are solutions to the system of VIP (1.4) as $x^*, y^* \in C$ iff the system of equation (3.2) has solutions $x^*, y^*, u^*, v^* \in H$, where

$$\begin{cases} x^* &= P_C(v^*), \\ y^* &= P_C(u^*), \end{cases} \quad (3.3)$$

$$\begin{cases} u^* &= x^* - \rho T x^*, \\ v^* &= y^* - \eta T y^*. \end{cases} \quad (3.4)$$

We suggest the following generalized system of fractional order Wiener - Hope Dynamical system as follows:

$$\begin{cases} D_t^{\alpha}x(t) &= \lambda_1\{P_C(y - \eta T(y)) - \eta T P_C(x - \rho T(x)) + \eta T(y) - x\}, \\ D_t^{\alpha}y(t) &= \lambda_2\{P_C(x - \rho T(x)) - \rho T P_C(y - \eta T(y)) + \rho T(x) - y\}, \end{cases}$$

(3.5)

which $x(t_0), y(t_0)$ in C, λ_1, λ_2 are constant with real positive t_0.

Theorem 3.3. Let C be the real Hilbert space H's closed and convex subset, which is non-empty. Consider a Lipschitz continuous mapping T with constant β from H to H. Then, for each $x_0, y_0 \in H$, generalized system of fractional order Wiener - Hope Dynamical system (3.5) has the exactly one continuous solutions, x(t), and y(t) with $x(t_0) = x_0$ and $y(t_0) = y_0$ over $[t_0, \infty)$.

Proof. let λ_1, λ_2 are two constants and the mapping G from cartesian product $H \times H$ to itself, define as follow:

$G(x, y) = (f(x), h(y))$,

where

$f(x) = \lambda_1\{P_C(y - \eta T(y)) - \eta T P_C(x - \rho T(x)) + \eta T(y) - x\}$,and

$h(y) = \lambda_2\{P_C(x - \rho T(x)) - \rho T P_C(y - \eta T(y)) + \rho T(x) - y\}$,

for all x and y in H. We may now specify the norm $\|\cdot\|_1$ on $H \times H$ by

$$\| (x, y) \|_1 = \| x \| + \| y \|, \forall (x, y) \in H \times H. \quad (3.6)$$

We known that $H \times H$ is a Hilbert space in regard to the norm $\|\cdot\|_1$. First, G is a Lipschitz continuous mapping, as we shall demonstrate. For this let $(x_1, y_1), (x_2, y_2) \in H \times H$. We have

$\| G(x_1, y_1) - G(x_2, y_2) \|_1 = = \| (f(x_1), h(y_1)) - (f(x_2), h(y_2)) \|_1$

$= \| (f(x_1) - f(x_2), h(y_1) - h(y_2)) \|_1$

$= \| f(x_1) - f(x_2) \| + \| h(y_1) - h(y_2) \|$

$= \| \lambda_1\{P_C(y_1 - \eta T(y_1)) - \eta T P_C(x_1 - \rho T(x_1)) + \eta T(y_1) - x_1\} - (\lambda_1\{P_C(y_2 - \eta T(y_2)) - \eta T P_C(g(x_2) - \rho T(x_2)) + \eta T(y_2) - x_2\}) \| + \| \lambda_2\{P_C(x_1 - \rho T(x_1)) - \rho T P_C(y_1 - \eta T(y_1)) + \rho T(x_1) - y_1\} - (\lambda_2\{P_C(x_2 - \rho T(x_2)) - \rho T P_C(y_2 - \eta T(y_2)) + \rho T(x_2) - y_2\}) \|$

$= \lambda_1 \| P_C(y_1 - \eta T(y_1)) - \eta(\mu F - T P_C(x_1 - \rho T(x_1)) + \eta T(y_1) - x_1 - P_C(y_2 - \eta T(y_2)) + \eta T P_C(x_2 - \rho T(x_2)) - \eta T(y_2) + x_2 \| + \lambda_2 \| P_C(x_1 - \rho T(x_1)) - \rho T P_C(y_1 - \eta T(y_1)) + \rho T(x_1) - y_1 - P_C(x_2 - \rho T(x_2)) + \rho T P_C(y_2 - \eta T(y_2)) - \rho T(x_2) + y_2 \|$

$\leq \lambda_1\{\| P_C(y_1 - \eta(\mu F - T(y_1)) - P_C(y_2 - \eta T(y_2)) \| + \eta \| TP_C(g(x_1) - \rho T(x_1)) - TP_C(x_2 - \rho T(x_2)) \| + \eta \| T(y_1) - T(y_2) \| + \| x_1 - x_2 \|\} + \lambda_2\{\| P_C(x_1 - sT(x_1)) - P_C(x_2 - \rho T(x_2)) \| + \rho \| TP_C(y_1 - \eta T(y_1)) - TP_C(y_2 - \eta(y_2)) \| + \rho \| T(x_1) - T(x_2) \| + \| y_1 - y_2 \|\}$

$\leq \lambda_1\{\| y_1 - \eta T(y_1) - (y_2 - \eta T(y_2)) \| + \eta\beta \| P_C(x_1 - \rho T(x_1)) - (P_C(x_2 - \rho T(x_2))) \| + \eta\beta \| y_1 - y_2 \| + \| x_1 - x_2 \|\} + \lambda_2\{\| x_1 - \rho T(x_1) - (x_2 - \rho T(x_2)) \| + \rho\beta \| P_C(y_1 - \eta T(y_1)) - (y_2 - \eta T(y_2)) \| + \rho\beta \| x_1 - x_2 \| + \| y_1 - y_2 \|\}$

$\leq \lambda_1\{\| y_1 - y_2 \| + \eta\beta \| y_1 - y_2 \| + \eta\beta\{\| x_1 - x_2 \| + \rho\beta \| x_1 - x_2 \|\} + \eta\beta \| y_1 - y_2 \| + \| x_1 - x_2 \|\} + \lambda_2\{\| x_1 - x_2 \| + \rho\beta \| x_1 - x_2 \| + \rho\beta\{\| y_1 - y_2 \| + \eta\beta \| y_1 - y_2 \|\} + \eta\beta \| x_1 - x_2 \| + \| y_1 - y_2 \|\}$

$\leq \lambda_1\{(1 + 2\eta\beta) \| y_1 - y_2 \| + (1 + \eta\beta + \eta\rho\beta^2) \| x_1 - x_2 \|\} + \lambda_2\{(1 + 2\rho\beta) \| x_1 - x_2 \| + (1 + \rho\beta + \eta\rho\beta^2) \| y_1 - y_2 \|\}$

$\leq \lambda_1\{(1 + 2\Phi\beta) \| y_1 - y_2 \| + (1 + \Phi\beta + \Phi^2\beta^2) \| x_1 - x_2 \|\} + \lambda_2\{(1 + 2\Phi\beta) \| x_1 - x_2 \| + (1 + \Phi\beta + \Phi^2\beta^2) \| y_1 - y_2 \|\} \leq \Delta\{(1 + 2\Phi\beta) \| y_1 - y_2 \| + (1 + \Phi\beta + \Phi^2\beta^2) \| x_1 - x_2 \|\} + \Delta\{(1 + 2\Phi\beta) \| x_1 - x_2 \| + (1 + \Phi\beta + \Phi^2\beta^2) \| y_1 - y_2 \|\} = \Delta(1 + 2\Phi\beta)\{\| x_1 - x_2 \| + \| y_1 - y_2 \|\} + \Delta(1 + \Phi\beta + \Phi^2\beta^2)\{\| x_1 - x_2 \| + \| y_1 - y_2 \|\} \leq 2\Delta(1 + 2\Phi\beta + \Phi^2\beta^2)\{\| x_1 - x_2 \| + \| y_1 - y_2 \|\} = 2\Delta(1 + \Phi\beta)^2\{\| x_1 - x_2 \| + \| y_1 - y_2 \|\} = 2\Delta(1 + \Phi\beta)^2\{\| (x_1 - x_2, y_1 - y_2) \|_1\},$

where $\Delta = max\{\lambda_1, \lambda_2\}, \Phi = max\{\rho, \eta\}$. Then G is Lipschitz continuous on$\|\cdot\|_1$. Hence for each point $(x_0, y_0) \in H \times H$, system (3.5) has precisely one continuous solution (x(t), y(t)), defined on $t \in [t_0, \Gamma)$ with the initial conditions $x(t_0) = x_0$ and $y(t_0) = y_0$.

Let $[t_0, \Gamma)$ be the maximum period of existence. Now, we prove that $\Gamma = \infty$. Under the assumptions made of T, the VIP (1.4) has unique solution, $x^*, y^* \in C$, with $x^* = P_C(y^* - \eta T(y^*)), y^* = P_C(x^* - \rho T(x^*))$,

Let x and y be arbitrary element of Hilbert space H. Then, we have

$\| G(x, y) \|_1 = \| (f(x), h(y)) \|_1 = \| f(x) \| + \| h(y) \| = \| \lambda_1\{P_C(y - \eta T(y)) - \eta TP_C(x - \rho T(x)) + \eta T(y) - x\} \| + \| \lambda_2\{P_C(x - \rho T(x)) - \rho(TP_C(y - \eta T(y)) + \rho T(x) - y\} \| = \lambda_1\{\| P_C(y - \eta T(y)) - x \| + \eta \| T(y) - TP_C(x - \rho T(x)) \|\} + \lambda_2\{\| P_C(x - \rho T(x)) - y \| + \rho \| T(x) - TP_C(y - \eta T(y)) \|\}$

$\leq \lambda_1 \| P_C(y - \eta T(y)) - x \| + \lambda_1\eta\beta \| y - (P_C(x - \rho T(x))) \| + \lambda_2 \| P_C(x - \rho T(x)) - y \| + \lambda_2\rho\beta \| x - (P_C(y - \eta T(y))) \|$

$= (\lambda_1 + \lambda_2\rho\beta) \| P_C(y - \eta T(y)) - x \| + (\lambda_2 + \lambda_1\eta\beta) \| P_C(x - \rho T(x)) - y \|$

$\leq (\Delta + \Delta\Phi\beta)\{\| P_C(y - \eta T(y)) - x \| + \| P_C(x - \rho T(x)) - y \|\}$

$\leq (\Delta + \Delta\Phi\beta)\{\| P_C(y - \eta T(y)) - P_C(y * - \eta T(y *)) \| + \| P_C(y * - \eta T(y *)) - x^* \| + \| x^* - x \| + \| P_C(x - \rho T(x)) - P_C(x^* - \rho T(x^*) \| + \| P_C(x^* - \rho T(x^*)) - y^* \| + \| y^* - y \|\}$

$\leq (\Delta + \Delta\Phi\beta)\{\| x^* - x \| + \| y^* - y \| + \| y - \eta T(y) - (y^* - \eta T(y^*)) \| + \| x - \rho T(x) - (x^* - \rho T(x^*)) \|\} \leq (\Delta + \Delta\Phi\beta)\{\| x - x^* \| + \| y - y^* \| + \| y - y^* \| + \Phi\beta \| y - y^* \| + \| x - x^* \| + \Phi\beta \| x - x^* \|\}$

$= (\Delta + \Delta\Phi\beta)(2 + \Phi\beta)\{\| x - x^* \| + \| y - y^* \|\} = (\Delta + \Delta\Phi\beta)(2 + \Phi\beta)\{\| x \| + \| x^* \| + \| y \| + \| y^* \|\} = (\Delta + \Delta\Phi\beta)(2 + \Phi\beta)\{\| x^* \| + \| y^* \|\} + (\Delta + \Delta\Phi\beta)(2 + \Phi\beta)\{\| x \| + \| y \|\} = (\Delta + \Delta\Phi\beta)(2 + \Phi\beta) \| (x^*, y^*) \|_1 + (\Delta + \Delta\Phi\beta)(2 + \Phi\beta) \| (x, y) \|_1,$

Hence,

$\| D_\omega^\alpha(x(t), y(t)) \| = \| G(x, y) \|_1 \leq k_1 + k_2 \| (x, y) \|_1$, (3.7)

where, $k_1 = (\Delta + \Delta\Phi\beta)(2 + \Phi\beta) \| (x^*, y^*) \|_1$ and $k_2 = (\Delta + \Delta\Phi\beta)(2 + \Phi\beta)$. Taking the fractional integral of (3.7), we get

$I_\omega^\alpha \| D_\omega^\alpha(x(t), y(t)) \| \leq I_\omega^\alpha[k_1 + k_2 \| (x, y) \|_1], \leq \frac{k_1}{\Gamma(\alpha)}\int_{t_0}^{t}(t - \tau)^{\alpha-1}d\tau + \frac{k_2}{\Gamma(\alpha)}\int_{t_0}^{t}(t - \tau)^{\alpha-1} \| (x(\tau), y(\tau)) \|_1 \, d\tau = \frac{k_1(t-t_0)^\alpha}{\Gamma(\alpha+1)}\frac{k_2}{\Gamma(\alpha)}\int_{t_0}^{t}(t - \tau)^{\alpha-1} \| (x(\tau), y(\tau)) \|_1$ (3.8)

Using Lemma 2.4 & 2.7, we get

$\| (x(t), y(t)) \| \leq \left\{\| (x(t_0), y(t_0)) \| + \frac{k_1(t - t_0)^\alpha}{\Gamma(\alpha + 1)}\right\} + \frac{k_2}{\Gamma(\alpha)}\int_{t_0}^{t}(t - \tau)^{\alpha-1} \| (x(\tau), y(\tau)) \|_1 \, d\tau$
$\leq \left\{\| (x(t_0), y(t_0)) \| + \frac{k_1(t-t_0)^\alpha}{\Gamma(\alpha+1)}\right\} exp\left\{\frac{k_2(t-t_0)^\alpha}{\Gamma(\alpha+1)}\right\}$, (3.9)

Hence, from (3.9) , Consequently, the solution is bounded on $[t_0, \infty)$. Therefore solution of generalize system of Wiener-Hopf dynamical system (3.5) is bounded on interval $[t_0, \Gamma)$, if Γ is finite. So, As a result, we say that $\Gamma = \infty$. Hence system of generalized fractional order Wiener-Hopf dynamical system (3.5) has exactly one continuous solution, x(t), y(t) with $x(t_0) = x_0$ and $y(t_0) = y_0$ over $[t_0, \Gamma]$. This complete the proof.

CONCLUSION

For the conventional system of variational inequalities, we have introduced and analysed the system of non-integer order Wiener-Hopf dynamical systems. The projection approach is devised and used to analyse these system of fractional dynamical systems connected to the system of variational inequalities. Under certain acceptable conditions, we have demonstrated that these fractional order Wiener-Hopf dynamical systems have only one solution to a system of VIP. Recurrent neural networks can be designed using the described dynamical systems to address variational inequalities and associated optimisation issues. Another potential direction for future work is to observe the stability of system of non-inter order Wiener-Hopf resolvent dynamical system and its application.

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